When we write expressions in exponential form, we won't have any radicals ($\sqrt{\ }$) in our final answers. We will use rational (fractional) exponents, if necessary, and we will make sure the exponents are positive in our final answers.

Example 1 – Write $\sqrt{m^{11}n^7}$ in exponential form.

Remember that square roots are the same thing as the exponent $^1/_2$.

$$\sqrt{m^{11}n^7} = (m^{11}n^7)^{1/2} = \boxed{\boldsymbol{m^{11/2}n^{7/2}}}$$

Example 2 – Write $\sqrt[3]{1000^2x^{-4}y}$ in exponential form.

Remember that cube roots are the same thing as the exponent $^1/_3$.

$$\sqrt[3]{1000^2x^{-4}y} = (1000^2x^{-4}y)^{1/3} = 1000^{2/3}x^{-4/3}y^{1/3} = \boxed{\frac{\mathbf{100}\boldsymbol{y^{1/3}}}{\boldsymbol{x^{4/3}}}}$$

Don't end with a negative exponent!

Example 3 – Write $\frac{1}{\sqrt[5]{w^{-3}}}$ in exponential form.

$$\frac{1}{\sqrt[5]{w^{-3}}} = \frac{1}{(w^{-3})^{1/5}} = \frac{1}{w^{-3/5}} = \boxed{\boldsymbol{w^{3/5}}}$$

Don't end with a negative exponent!

When we multiply and divide radical expressions, we need to

1) Rewrite the radical expressions in exponential form (using fractions for exponents, if necessary).
2) Make sure the bases are the same. If they aren't, then take action to get them to be the same numbers (are they powers of the same number?).
3) Multiply and/or divide using exponent rules.
4) Pay attention to whether the final answers are to remain in exponential form or if they need to be changed back to radical form.

Example 4 – Express $\sqrt[10]{32} \bullet \sqrt[8]{4}$ in simplest radical form

Step 1- Rewrite in exponential form

$$\sqrt[10]{32} \bullet \sqrt[8]{4} = 32^{1/10} \bullet 4^{1/8}$$

Step 2- Get same bases

Our bases are not the same. One of them is 32 and the other one is 4. Are these numbers (32 and 4) powers of the same number? Yes! We know 32 (2^5) and 4 (2^2) are both powers of the number **2**. We will rewrite our expression so the bases are both **2**.

$$32^{1/10} \bullet 4^{1/8} = (2^5)^{1/10} \bullet (2^2)^{1/8} = \mathbf{2}^{5/10} \bullet \mathbf{2}^{2/8} = 2^{1/2} \bullet 2^{1/4}$$

Step 3- Multiply/divide using exponent rules

$$2^{1/2} \bullet 2^{1/4} = 2^{1/2 + 1/4} = 2^{2/4 + 1/4} = 2^{3/4}$$

Step 4- Exponential or radical form?

We were asked for radical form. The 4th root of 2 is an irrational number, so we'll have to do the power of 3 first and take the 4th root last.

$$2^{3/4} = (2^3)^{1/4} = 8^{1/4} = \sqrt[4]{8}$$

$$\boxed{\sqrt[4]{\mathbf{8}}}$$

<u>Example 5</u> – Express $\sqrt[3]{4} \bullet \sqrt[4]{8}$ in simplest exponential form

Step 1- Rewrite in exponential form

$$\sqrt[3]{4} \bullet \sqrt[4]{8} \;=\; 4^{1/3} \bullet 8^{1/4}$$

Step 2- Get same bases

Our bases are not the same. One of them is 4 and the other one is 8. Are these numbers (4 and 8) powers of the same number? Yes! 4 (2^2) and 8 (2^3) are both powers of the number **2**. We will rewrite our expression so the bases are both **2**.

$$4^{1/3} \bullet 8^{1/4} \;=\; (2^2)^{1/3} \bullet (2^3)^{1/4} \;=\; \mathbf{2}^{2/3} \bullet \mathbf{2}^{3/4}$$

Step 3- Multiply/divide using exponent rules

$$2^{2/3} \bullet 2^{3/4} \;=\; 2^{2/3 + 3/4} \;=\; 2^{8/12 + 9/12} \;=\; 2^{17/12}$$

Step 4- Exponential or radical form?

We were asked for exponential form, so we are done.

$$\boxed{\mathbf{2^{17/12}}}$$

Example 6 – Express $\sqrt{x} \bullet \sqrt[3]{x} \bullet \sqrt[6]{x}$ in simplest exponential form

Step 1- Rewrite in exponential form

$$\sqrt{x} \bullet \sqrt[3]{x} \bullet \sqrt[6]{x} = x^{1/2} \bullet x^{1/3} \bullet x^{1/6}$$

Step 2- Get same bases
Our bases are the same – every one of them is x.

Step 3- Multiply/divide using exponent rules

$$x^{1/2} \bullet x^{1/3} \bullet x^{1/6} = x^{1/2 + 1/3 + 1/6} = x^{3/6 + 2/6 + 1/6} = x^{6/6} = x^1 = x$$

Step 4- Exponential or radical form?
We were asked for exponential form, but it doesn't really matter this time as there aren't any radicals or roots at the end.

$$\boxed{\boldsymbol{x}}$$

<u>Example 7</u> – Express $\left(x^{3/2} - 2x^{5/2}\right) \div x^{1/2}$ in simplest exponential form

Step 1- Rewrite in exponential form
It is already written in exponential form.

Step 2- Get same bases
Our bases are the same – every one of them is x.

Step 3- Multiply/divide using exponent rules
When we *multiply* expressions with the same bases, we *add exponents*.
When we *divide* expressions with the same bases, we *subtract exponents*.
(Dividing by $x^{1/2}$ is the same as multiplying by $x^{-1/2}$)

$$\left(x^{3/2} - 2x^{5/2}\right) \div x^{1/2} = x^{3/2 - 1/2} - 2x^{5/2 - 1/2} = x^{2/2} - 2x^{4/2} = x - 2x^2$$

Step 4- Exponential or radical form?
We were asked for exponential form, but it doesn't really matter this time as there aren't any radicals or roots at the end.

$$\boxed{\boldsymbol{x - 2x^2}}$$